Individual contest - day 2

youngest group

Tuesday, 25 September 2007

21. Find all integer solutions in triples (x, y, z) of equation:

$$x^3 + 2y^3 = 4z^3$$

22. Show that for nonnegative a, b, c an inequality holds:

$$a^3 + b^3 + c^3 + a^2b + b^2c + c^2a \geqslant 2(ab^2 + bc^2 + ca^2)$$

23. A polygon $A_1A_2A_3...A_n$ is given. Construct such points B_1, B_2, ..., B_n, for which points A_1, A_2, ..., A_n are midpoints of segments B_1B_2, B_2B_3, ..., $B_{n-1}B_n$, B_nB_1 respectively.

24. Quadrilateral $ABCD$ is inscribed in a circle with center O. Diagonals AC and BD intersect at P. Circumcircles of triangles ABP and CDP intersect at points P and Q. Show that if points P, Q and O are pairwise distinct then $\angle PQO$ is right.

25. Skrzypen doesn't want to grant his 37 Skrzypusies access to his private movie resources. Therefore he has constructed a safe and put a movie „The Pirates" inside. Now Skrzypen would like to add a number of locks to the safe and give every Skrzypus keys to some of them. The distribution of keys should follow conditions:

(a) every 36 Skrzypusies are able to open the safe,

(b) no 35 Skrzypusies are able to open the safe.

What is the minimal number of locks Skrzypen has to assemble?

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younger group

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26. Is it possible to assign one of 2007 colors to every point of a plane in such a way that in every closed disk there are points of all the colors?

Individual contest - day 2

older group

Tuesday, 25 September 2007

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27. A sequence $\{a_n\}$ of positive integers satisfies following conditions:

$$a_1 = 2, a_n = \lfloor \frac{3a_{n-1}}{2} \rfloor$$

Show that in this sequence there is infinitely many even numbers and infinitely many odd numbers.

28. Does there exists a sequence $a_1, a_2, a_3...$ of real numbers and non-constant polynomial $P(x)$ such that equality $a_m + a_n = P(mn)$ holds for all positive integers m, n?

29. Is it possible to assign one of 2007 colors to every point of a plane in such a way that on every circle there are points of all the colors?

Individual contest - day 2

oldest group

Tuesday, 25 September 2007

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210. Determine the smallest odd integer $n \geqslant 1$ satisfying following condition: there exist infinitely many integers, which squares are equal to sum of squares of n consecutive integers.

211. Show that for every positive integer n:

$$\sum_{i+j=n} \binom{2i}{i}\binom{2j}{j} = 4^n$$