

Sumy

1. Oblicz $\sum_{i=0}^n \binom{n}{i}$.
2. Oblicz $\sum_{i=0}^n \binom{n}{i} (-1)^i$.
3. Oblicz $\sum_{i=0}^n \binom{n}{i}^2$.
4. Oblicz $\sum_{i=0}^n \binom{n}{i}^2 (-1)^i$.
5. Oblicz $\sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{i} \binom{n-i}{i} 2^{n-2i}$.
6. Oblicz $\sum_{i=0}^n \binom{n}{i} k^i$.
7. Oblicz $\sum_{i=1}^n i(i-1)(-1)^i$.
8. Oblicz $\sum_{i=0}^n i \binom{n}{i}$.
9. Oblicz $\sum_{i=0}^n \frac{\binom{n}{i}}{i+1}$.
10. Oblicz $\sum_{i=1}^n (n-i+1) \binom{n}{i} \binom{n}{i-1}$.
11. Oblicz $\sum_{k=0}^n \sum_{l=0}^{n-k} \binom{n}{k} \binom{n-k}{l}$.
12. Niech n, p, r całkowite dodatnie takie, że $n \geq p + r$. Oblicz $\sum_{i=0}^n \binom{i}{p} \binom{n-i}{r}$.
13. Pokazać, że dla każdego całkowitego dodatniego n podzielnego przez 3 zachodzi:

$$\sum_{k=0}^n \sum_{i=0}^n \binom{n}{k} \binom{n}{i} \binom{n-k}{n-2k-i} (-1)^{n-i} = \binom{n}{\frac{n}{3}}$$