Team contest

younger group

Wednesday, 26 September 2007

31. Points C and D lie on semicircle with diameter AB, and C is closer to A than D. Let M,N,P be midpoints of segments AC, BD and CD respectively. Point O_A is circumcentre of triangle ACP, and point O_B is circumcentre of triangle BDP. Show that $O_AO_B||MN$.

32. Show that for integer $n \geqslant 2$ an inequality holds:

$$n(\sqrt{n+1}-1) \leqslant 1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n} \leqslant 1+n(1-\frac{1}{\sqrt[n]{n}})$$

33. Two circles o_1 i o_2 intersect at points X and Y. Line l is tangent to o_1 and o_2 in points A and B respectively. Let m be a line tangent in X to circumcircle of triangle XAB and n a line tangent in Y to circumcircle of triangle YAB. Show that lines l, m and n have a common point.

34. Show that for positive a, b, c an inequality holds:

$$3(a+b+c) \geqslant 8\sqrt[3]{abc}+\sqrt[3]{\frac{a^3+b^3+c^3}{3}}$$

35. Show that if p is an odd prime number and $k \geqslant p$ then:

$$\binom{k}{p} \equiv \left\lfloor \frac{k}{p} \right\rfloor \mod p$$

36. Positive irrational numbers p, q satisfy condition $\frac{1}{p}+\frac{1}{q}=1$. Show that for all $n \in \mathbb{N}$ exists such $k \in \mathbb{N}$ that $n = \lfloor pk \rfloor$ or $n = \lfloor qk \rfloor$.

37. A number of schools took part in a tournament. Each of them was represented by a team consisting of girls and boys. The total number of girls taking part in the tournament differed from the total number of boys by 1. Then each two contestants from different schools played a match. The number of matches between contestants of different sex differed from the number of matches between contestants of the same sex by 1. Determine the maximal possible number of participating schools represented by an odd number of contestants.

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38. Circles o_1 and o_2 intersect at points A and B. Let l be a line tangent to o_1 and o_2 in points X and Y respectively. Lines tangent in points X and Y to a circumcircle of triangle AXY intersect at point C. Let B' be a symmetrical reflection of point B through line l. Show that points B',A and C are collinear.

39. On a plane 9 points are given, no 3 of them collinear. Show, that for every of these points the number of triangles with vertices in other 8 points, to which interior this point belongs, is even.

310. Let $ABCD$ be a parallelogram. Line l, passing through point A intersects rays BC and DC at X and Y respectively. Let K and L be centers of circles outerscribed to triangles ABX and ADY, tangent to sides BX and DY respectively. Show, that for given parallelogram $ABCD$ the measure of angle KCL does not depend on the choice of line l.

311. Find the set of values of $x_0, x_1 \in \mathbb{R}$ for which in sequence $(x_n)_{n \in N}$ defined by condition

$$x_{n+1} = \frac{x_{n-1}x_n}{3x_{n-1} - 2x_n}, \qquad n \geqslant 1$$

there is infinitely many integer numbers.