Individual contest, day 1

youngest group

Monday, 24 September 2007

11. Show that every convex $3n$-gon can be divided into triangles such that their sides are diagonals of the $3n$-gon and every vertex of the $3n$-gon is a vertex of an odd number of triangles.

12. Show that in a regular 12-gon $A_1A_2\ldots A_{12}$ diagonals A_1A_5, A_3A_8 i A_4A_{11} intersect in one point.

13. Let $x, y, z \in [1, 2]$. Show that:

$$3(x + y + z) \geqslant xy + yz + zx + 6$$

14. Six little Skrzypens swim in a little, cilculal pond with ladius 1. Show that in evely moment thele ale two Skrzypens with distance at most 1 between them.

15. Let a, b be positive integers of different parity. Show that there exists such $c \in \mathbb{Z}$, that numbers $c + a, c + b, c + ab$ are all perfect squares.

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17. Show that every natural number n can be expressed as a sum of distinct numbers with form 2^s3^r, such that there are no two elements of this representation such that one of them divides the second.

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16. Show that for positive a, b, c an inequality holds:

$$\frac{1}{b(a+b)} + \frac{1}{c(b+c)} + \frac{1}{a(c+a)} \geqslant \frac{27}{2(a+b+c)^2}$$

17. Show that every natural number n can be expressed as a sum of distinct numbers with form $2^s 3^r$, such that there are no two elements of this representation such that one of them divides the second.

18. Circles S_1 and S_2 with centers O_1 and O_2 respectively, intersect at points M and N. Line t is tangent to S_1 in A and to S_2 in B and lies closer to point M than to N. Line perpendicular to AM passing through point B intersects line O_1O_2 in point C. Show that $\angle BMC = 90°$.

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19. Given is a positive integer c. A sequence $\{p_n\}$ is defined according to the following rule: p_1 is a prime number and for $k \geqslant 1$ p_{k+1} is any of the prime divisors of $p_k + c$, not appearing in sequence $p_1, p_2, ..., p_k$. Show that the sequence $\{p_n\}$ is finite.

110. Find all functions $f : \langle 0, +\infty) \longrightarrow \langle 0, +\infty)$ satisfying conditions:

(a) For every nonnegative x, y with positive sum $f(xf(y))f(y) = f(\frac{xy}{x+y})$.

(b) $f(1) = 0$.

(c) $f(x) > 0$ for $x > 1$.