Individual contest, day 0

youngest group

Sunday, 23 September 2007

1. Two hundred contestants took part in Mathematical Olympiad. They were asked to solve 6 problems. It is known, that each problem has been solved by at least 120 contestants. Show that there are such two contestants, that every problem has been solved by at least one of them.

2. How many (at most) disjoint squares 2×2 man can places on the square chessboard $n \times n$ in such a way that the sides of the squares were situated along the lines dividing grids?

3. Positive numbers $a_1, a_2, \ldots, a_n$ sum up to 1. Show that an inequality holds:

$$a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n \leqslant \frac{1}{4}$$

4. A square $ABCD$ is given. Points E and F lay on the sides AB and BC respectively and $BE = BF$. Point S is a perpendicular projection of point B onto line EC. Show that an angle DSF is right.

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6. Find the number of subsets M of the set $\{1, 2, \ldots, 36\}$ satisfying following condition:

For every two distinct $x, y \in M$ *also* $|x - y| \in M$.

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5. In acute-angled triangle ABC bisector of $\angle ABC$ intersects side AC in point D. Points E and F are perpendicular projections of points A and C onto line BD respectively. Point M is a perpendicular projection of point D onto line BC. Show that $\angle EMD = \angle FMD$.

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7. Show that for positive a, b, c, d, e an inequality holds:

$$\frac{a}{e+a+b} + \frac{b}{a+b+c} + \frac{c}{b+c+d} + \frac{d}{c+d+e} + \frac{e}{d+e+a} \leqslant 2$$

8. Show that there are two squares of consecutive positive integers, such that between them there are at least 2007 prime numbers.